

DTR RETICLE AND BALLISTICS APP USES THE FOLLOWING..

SmartShot Is Accurate At Extreme Long Ranges

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SmartShot Is Accurate From Mach 3 to Mach 0.6

The SmartShot ballistic app developed by ProjectileScience Inc. accurately models long range trajectories of all VLD bullets we have tested from Mach 3 out to extreme long ranges where the Mach number has dropped to 0.6. Trajectories computed with five of the most well respected ballistics apps were compared to our actual trajectories for several VLD bullets from a 115 gn 6mm to a 364 gn 375 caliber. All ballistic apps we tested were accurate in most of the supersonic regime but none match our actual measured trajectories in the subsonic regime. The differences are hundreds of inches as shown in the Path Difference graph on page 4. We believe that SmartShot works in both the supersonic and the subsonic regimes because the math model is better than the traditional Bashforth/Ingalls/Sacci model although there may be other reasons. The only significant operational difference users will notice between SmartShot and traditional ballistic apps is SmartShot replaces ballistic coefficients, form factors and drag functions with a new, and we think better, drag metric, velocity retention. This paper describes the characteristics of SmartShot and velocity retention. Fully functional beta versions of SmartShot and a detailed users' manual are available free at <https://projectilescience.com/>.

Note

We note here at the outset that we were puzzled that all of the five well respected ballistic apps we studied showed substantially more subsonic drop than we measured and which SmartShot predicts. We continue to search for some explanation other than the easy one - that the other apps do not work in the subsonic regime because they were never intended to do so. A second explanation might be that these apps have not been checked against actual trajectories as far out as Mach 0.6. A third possible explanation for the difference is that all data with which SmartShot was developed was shot either by David Tubb or myself under carefully controlled conditions. We minimized to the extent possible all error sources which could produce point of impact dispersion. All data were shot with Tubb rifles fitted with Schneider 5P fast twist barrels and scopes with Dynamic Targeting Reticles. Perhaps the most important characteristic of our rifles was that the fast twist barrels, from 7 to 8 inches per turn, which increased the gyroscopic stability substantially. It is becoming apparent that subsonic stability requires greater gyroscopic stability than the traditional value of 1.4 at the muzzle.

Subsonic instability would increase subsonic drag which would increase drop. Thus the increased drop computed by the five ballistic apps in the subsonic regime might correct for bullets with traditional values of gyroscopic stability. We believe SmartShot is correct for bullets with increased stability. I hope that shooters will try SmartShot in the subsonic regime and let me know what they find.

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Summary

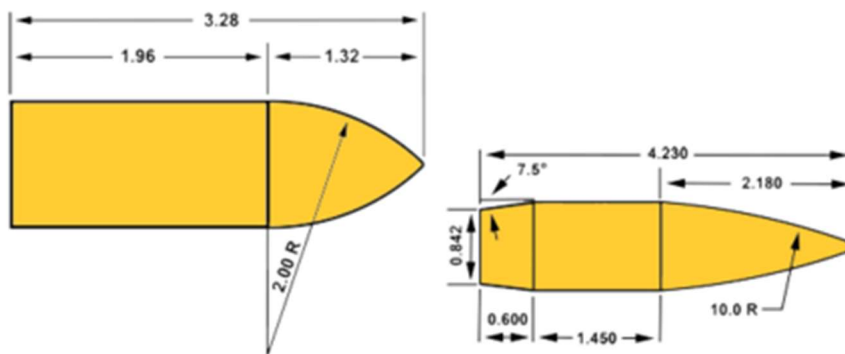
1. The SmartShot ballistic app was developed by ProjectileScience Inc. specifically for David Tubb's Dynamic Targeting Reticule (see: https://www.davidtubb.com/index.php?route=account/download/free&download_id=30,) but it also supports traditional grid and dial reticles. SmartShot is a one page spreadsheet that will run on any browser, both on-line and off-line. Fully functional beta versions are available free at <https://projectilescience.com/products/>
2. We analyzed Applied Ballistics, Ballistic Explorer, Hornady 4DoF, JBM and Sierra Infinity 7 plus several others. All matched our actual trajectories in most of the supersonic regime but none matched our actual trajectories in the subsonic regime; at 2,000 yards the errors were from -40 to -140 inches and at 2,400 yards, -150 to -350 inches. See the Path Difference graph on page 4.
3. We have validated SmartShot with rigorous field testing for nine years with bullets ranging in size from 115gn 6mm DTAC bullet out to 2400 yards to the 364gn 375 Warner Flat Line bullet out to 3200 yards. SmartShot works with every VLD bullet we have tested out to approximately Mach 0.6. There is no indication that Mach 0.6 is the SmartShot limit but we do not yet have data at lower Mach numbers. The SmartShot ballistic model was the basis for the Dynamic Targeting Reticule (DTR) App used by David Tubb and Nate Stallter in their 2018 ELR records. Video clips on the ProjectileScience web site demonstrate the successful use of SmartShot against small dynamic targets out to 2,000 yards in Africa. See <https://projectilescience.com/tech-notes/>, Tech Note TN181118, Study of Baboon Behavior and Reaction Timing.
4. SmartShot utilizes a unique analytical solution to the ballistic differential equation developed by Art Pejsa and described in his book, *New Exact Small Arms Ballistics*. His solution enables tailoring the analytical trajectory equation to precisely match the actual trajectory in the supersonic regime. ProjectileScience enhanced Pejsa's solution to support subsonic trajectories. The trajectory of every VLD bullet we tested was accurately described by the resulting single general ballistic equation used by SmartShot.
5. Pejsa, recognizing the well known limitations of the Brashforth\Ingalls\Siacci model using ballistic coefficients, form factors and drag functions, developed not only an analytical solution but a new drag metric which mathematically quantifies the ability of the bullet to retain velocity. He named it "retard coefficient"; a directly measurable bullet characteristic. SmartShot uses a slightly different but functionally identical metric we named "velocity retention" which is simply the distance from the muzzle at which the bullet has lost 1% of velocity. With the exception of the drag metric, the fundamental differences between traditional ballistic apps and SmartShot are invisible to the users - except for the accuracy of the results especially in the subsonic regime.
6. In addition to calculation of the basic trajectory, SmartShot includes optional capabilities to compute the effects of spin drift, Coriolis, crosswind jump, differential wind drift, powder temperature sensitivity, crosswind boundary layer, obstacle clearance distance and others. A detailed manual is provided at <https://projectilescience.com/manuals-2/>

Five Ballistic Apps Studied Do Not Match Our Actual Subsonic Trajectories

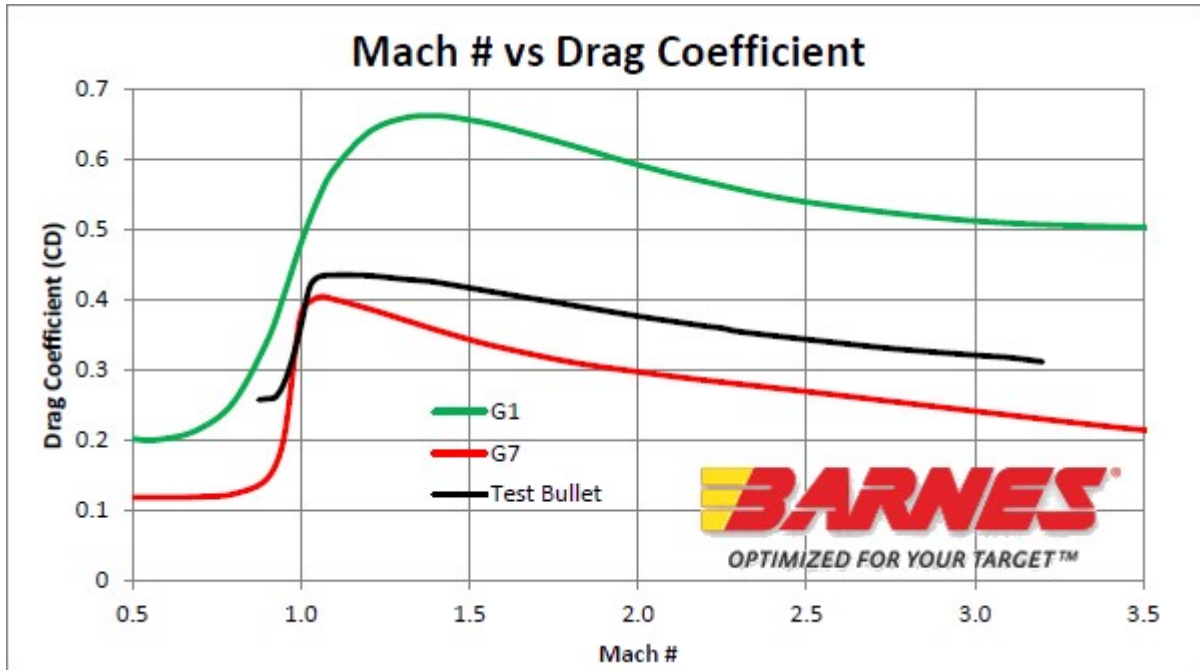
Most traditional ballistic apps rely on ballistic coefficients and thus form factors and drag profiles to account for the change of the drag coefficient with Mach number as shown by

the drag curve on page 4 which was copied from the Barnes web site. The traditional ballistic model which originated in England and was subsequently improved during more than a century has met the needs of most of the sport shooting community since the 1870's. However, in recent years shooters have begun to shoot at longer ranges. Long range shots on game of 800 to 1,000 yards are now shown on TV hunting shows. NRA competitions at the Whittington Center have targets at two miles. The Extreme Long Range competitions now set the long range marksmanship standard because of the prohibition of sighters thus requiring for the first time the ability to call long supersonic and perhaps even subsonic firing solutions. Consequently, our testing indicates that existing ballistic apps may be inadequate for today's long range shooters.

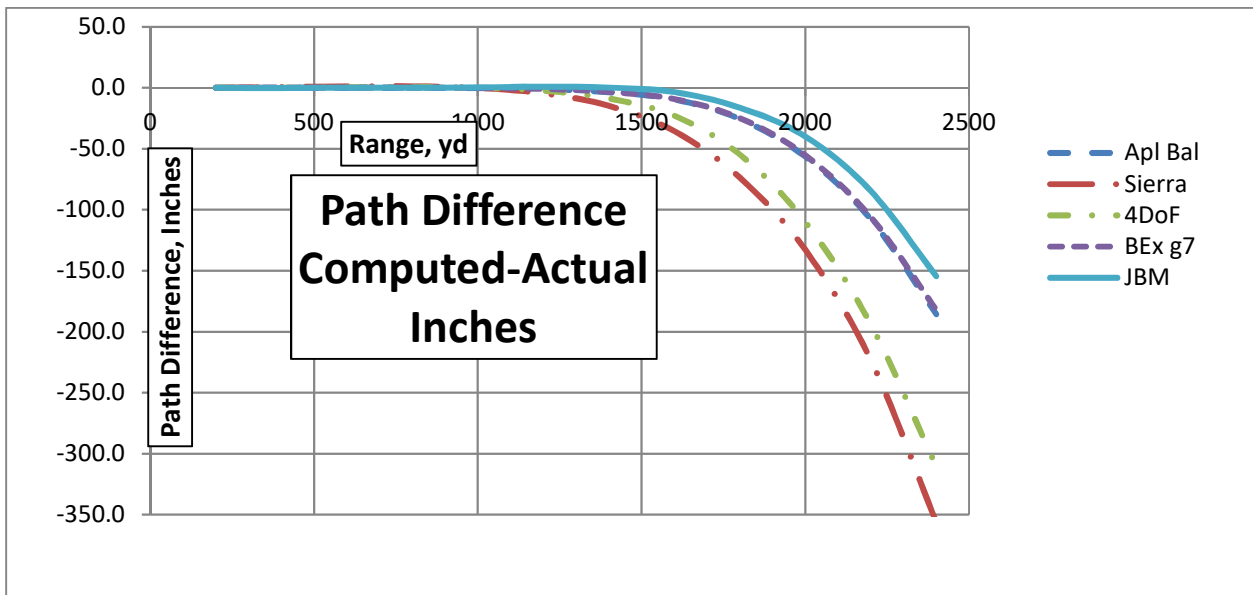
Traditional ballistic apps were developed from and are still mostly based on a standard projectile with a fixed size, shape and weight like the one below. The following sketches were copied from Wikipedia.



The drag curves of these obsolete projectiles were extrapolated to modern VLD bullets. Adjustments for weight, diameter, shape and Mach number are of course required. But even given these adjustments, it would be astounding if drag data for such a primitive projectile could be extrapolated with acceptable accuracy to a modern VLD bullet without using several different ballistic coefficient's over most of the supersonic regime, never mind the subsonic regime. Barnes Bullets makes this point with the following graph which shows the dramatic difference between the G1 and G7 ballistic coefficient profiles and an actual test bullet. Shooters can "trim" the traditional ballistic apps to correct the errors at a specific range or perhaps a usable range interval but not all ranges, especially not into the subsonic regime - because the trajectory *shapes* are different than a real bullet. Bullet-specific custom drag profiles are beginning to be offered to mitigate this problem.



We evaluated five of the most respected ballistic apps by comparing analytical trajectories to actual trajectories of David Tubb's 115gn 6xc bullet out to 2,400 yards. The path difference was computed by subtracting the actual path from the analytical path computed by each ballistic app. Since we are examining here the *shape* of the path curve not the exact *value* of the path at any specific range, each trajectory was normalized so that all would have the same path value at 1,000 yards. SmartShot was shaped during years of development to match the actual trajectories. So SmartShot was the reference used to compute the path difference. More on SmartShot later.



The graph shows all five ballistic apps have good accuracy in most of the supersonic regime but none match our actual trajectories in the subsonic regime. SmartShot was developed to match our actual trajectories as far out as we have data which is 2,400 yards for the 115 gn DTAC bullet and at 3200 yards for the Warner 364 gn 375 Flatline bullet. We believe SmartShot will prove to be accurate out to Mach 0.6 which will be about 4,400 yards for the Warner 364 gn 375 Flat Line bullet.

We Believe The SmartShot Math Model Is More Accurate

Art Pejsa (Circa 1920 - 2014, a WWII B-29's pilot and polymath) found an ingenious analytical solution for the equation governing ballistic flight with drag under the influence of gravity. He published many articles in *Precision Shooting* and other magazines but his most complete presentation, so far as we know, is in his book, *New Exact Small Arms Ballistics*, © 2008. Pejsa recognized that the bullet trajectory, ignoring for the moment wind, spindrift, etc., depends on only two forces: 1) gravity pulling the bullet down and 2) aerodynamic drag slowing the bullet. The equation describing the effect of gravity is well known so that part was easy. Describing the effect of drag, which is a bit harder, drove the 19th century ballistic engineers to a simple extrapolation of the drag observed on a "standard" shaped projectile which lead directly to the Bashforth\Ingalls\Sacci model.

Pejsa, working 100 years later, recognized that this traditional ballistic model was no longer adequate for improved long range rifle systems. His equation, as shown in his book, has factors with which allow the analytical trajectory to be precisely matched to the actual trajectory. Pejsa worked primarily in the supersonic regime but ProjectileScience used the same equation with different coefficients to describe subsonic trajectories. SmartShot uses the same equation with two separate sets of drag metrics linked at transition to describe first the supersonic trajectory and then the subsonic trajectory.

Using thousands of carefully controlled and measured shots since 2009 we have proved that SmartShot accurately describes the trajectory of all VLD bullet we have tested from the muzzle to Mach 0.6 using only two bullet-specific drag metrics, one in the supersonic regime and one in the subsonic regime. To our surprise, the coefficients used to shape the trajectory are not bullet-specific for any VLD bullet we tested. In addition to the usual input variables SmartShot needs only two field measurements: 1. the path at a range near the transition and 2. the path near Mach 0.6 or near the longest range of interest. Getting good path data is harder than it sounds because of the real world of wind and pointing errors and from a practical point of view, simply getting point of impact data from a long range target. Time-of-flight data is of course better than path data but the instrumentation is more expensive, complex and thus not readily available to most shooters. We used both path and time-of-flight data cross-checked at many locations at different altitude, temperature and relative humidity, different rifle systems and different shooters since 2009.

Pejsa Replaced Ballistic Coefficients, Form Factors and Drag Profiles With One Equation

In his book, Pejsa shows the development of his analytical solution starting with the most basic observations: the bullet must continually trade velocity for distance. During the flight the aerodynamic drag force slows the bullet down so:

Aerodynamic Drag = Bullet Deceleration

Aerodynamic drag is computed in the usual way:

$$\text{Aero Drag Force} = 1/2 \times \text{air density} \times \text{velocity}^2 \times \text{area} \times \text{drag coefficient}$$

The bullet deceleration is:

$$\text{Deceleration Force} = \text{mass} \times \text{acceleration}$$

Then he replaced acceleration with the rate at which velocity changes with time, dV/dT :

$$\text{Deceleration Force} = \text{mass} \times d\text{Velocity}/d\text{Time}$$

Then he set the drag force equal to the deceleration force:

$$1/2 \times \text{air density} \times \text{velocity}^2 \times \text{area} \times \text{drag coefficient} = \text{mass} \times d\text{Velocity}/d\text{Time}$$

While the bullet is trading velocity for distance, it is continuing to accelerate downward under the influence of gravity which is a constant acceleration. The problem is that the bullet velocity is changing very rapidly. The aerodynamic drag force at the muzzle is 50 to 70 times greater the force of gravity. Not only is the velocity changing non-linearly, the drag coefficient changes non-linearly with velocity. So it is a complex problem.

There is nothing new or different about Pejsa's approach at this point. This is the classical derivation but this is where the math becomes difficult. Pejsa did what few, perhaps no others have accomplished; he developed an accurate approximate analytical solution to the ballistic problem by introducing a number of ingenious simplifying assumptions. Our tests have shown his approximate solution is as accurate as we can measure.

One of Pejsa's several innovations was a new drag metric that quantitatively describes the bullet's ability to trade velocity for distance - with one simple, easily measured number. He called it "retard coefficient". SmartShot uses virtually the same metric renamed "velocity retention", changing the emphasis from retardation to retention. The velocity retention value is a real number normally between 20 and 100 that is the number of feet of bullet flight during which the projectile loses 1% of velocity, or said another way, retains 99% of velocity. So a velocity retention value of 60 means it retains 99% of velocity over a distance of 60 feet, e.g., it gives up 1 % of velocity in 60 feet. A heavy, low drag bullet with a high velocity retention value will carry velocity further than a light, high drag bullet with a low velocity retention value. Compare the simplicity of velocity retention to the tortured path from Bashforth's standard projectile through Ingalls' and Sacci's work, all of which eventually washes ashore in the form used in most of today's ballistic apps. To belabor the point, compare the complexity of McDonald and Almgren's article *The Ballistic Coefficient* to the simple definition of velocity retention in this paragraph.

Pejsa pointed out that velocity retention can be measured with two chronos several hundred yards apart. However, in order to improve the accuracy we measured the supersonic *path* at two points: 600 yards and 1000 yards. So with three points, the muzzle, the mid-point (600 yards) and the end point (1000 yards), we established the trajectory of the bullet. Given the physical similarity of VLD bullets, these three points describe a unique trajectory *shape*: physics does not permit any other well behaved trajectory that will intersect these three points. Then by trial and error we forced the *shape* of the analytical trajectory to match the actual trajectory. Then, again by trial and error, we found the velocity retention value that caused the analytical trajectory to exactly match the actual trajectory.

This was a slow process that would probably have doomed the concept to one of academic interest if the iterative process had to be repeated for every bullet. But eventually we observed that every VLD bullet we tested has the same trajectory *shape*. The only bullet-specific variable is the velocity retention value which is easily measured. For precision long range shooting, the actual trajectory is a function not only of the bullet but of the entire rifle system because the behavior of the bullet at the muzzle strongly effects the bullet trajectory which can vary from rifle to rifle depending perhaps primarily on the barrel but also on the stock flexibility and support. So for maximum long range accuracy each individual rifle system must be individually calibrated. Having said that, I should add that we found that all of our rifle systems, all of which were Tubb systems, produced the same ballistic results. Rifles with flexible stocks and low quality barrels and scopes are likely to produce variable results.

We repeated this process in the subsonic regime and added a linkage from the supersonic regime to subsonic regime at the transition. We then had developed an approximate analytical solution for the complete trajectory from the muzzle through transition and out to about mach 0.6, which is about 2400 yards for the 115 gn 6xc and 4,400 yards for the 364 gn 375 Flat Line that is as accurate as we can measure.

Velocity Retention Values

The following table shows the supersonic and subsonic velocity retention values of several well known bullets. Supersonic velocity retention is VRsup; subsonic is VRsub. Note that the flatter shooting bullets have larger velocity retention values. All ballistic coefficient values except the Warner Flatline are from Litz's book; Flatline is from the Warner web site.

Bullet	VRsup	VRsub	BCg1	BCg7
223, 77 gn, Sierra Match King	37	100	0.371	0.190
308, 175 gn, Sierra Match King	45	120	0.475	0.243
243, 115 gn, DTAC (similar to Sierra Match King)	64	130	0.540	0.276
338, 300 gn, Sierra Match King	76	120	0.745	0.381
375, 364 gn, Warner Flatline	99	135	0.980	0.474

Our web site lists the velocity retention values for more than 50 popular VLD, and near-VLD, bullets. SmartShot can convert both BCg1 and BCg7 to approximate velocity retention values. See <https://projectilescience.com/manuals-2/>, 1890814 Bullet VR's

Ballistic Coefficients and Velocity Retention Are Completely Different

While ballistic coefficients and velocity retention are both drag metrics, they are completely different. Ballistic coefficients have the units of pounds per square inch while velocity retention is in feet.

Not all experts agree that Pejsa's model and velocity retention are useful. For an opposing view, see: B. Litz, *Applied Ballistics for Long-Range Shooting*, pg 104. Other's have posted similar opinions. But despite the expert opinions, SmartShot's application of Pejsa's math model is as accurate as we can measure. Trajectories computed with the five ballistic programs we analyzed do not match our actual data. As noted in the introduction there may be several reasons for this discrepancy.

Conclusions

1. SmartShot trajectories match all VLD bullets we tested in both the supersonic and subsonic regimes.
2. Most of the ballistic apps matched our data out to about 800 yards. Some matched through most of the supersonic regime. None matched our actual trajectories in the subsonic regime.
3. We believe that Pejsa's ballistic solution is fundamentally a better ballistic model than the traditional Bashforth/Ingalls/Sacci model which relies on a standard projectile model and requires the complex trail of ballistic coefficients, form factors and drag profiles (G1, G7, etc.).
4. We believe that SmartShot's drag metric, velocity retention, matches actual trajectories better and is easier to understand and use.
5. We believe the reason SmartShot works for many VLD bullets is that the trajectory of every VLD bullet has the same fundamental *shape*. The ability of different bullets to retain velocity differently is mathematically quantified by the drag metric, velocity retention.
6. Beta versions of SmartShot are available free at <https://projectilescience.com/products/>. All advanced capabilities are functional: spin drift, Coriolis, crosswind jump, differential wind drift, powder temperature sensitivity, crosswind boundary layer, obstacle clearance distance and others. SmartShot is a one page spreadsheet which can be used both on-line and off-line and on work stations, tablets and cell phones.

Appendices

1. There Are Three Primary Versions of SmartShot

As noted earlier, SmartShot was developed specifically for David Tubb's Dynamic Targeting Reticle of which there are three basic versions plus several special versions; 6mm 6xc DTAC, 308 Winchester and the 223 Remington plus special versions for short barreled rifles, etc. However, the focus of this paper is the 6xc reticle which has shown to be applicable to all VLD bullets we have tested.

2. Velocity Retention Dates Back to 1848

It's interesting to note that retention of velocity was recognized as a fundamental exterior ballistics parameter long before Brashford's work in 1870. William Greener, in his book, *The Science of Gunnery As Applied To Military and Sporting Arms*, published in 1846, used the

concept of "retention of velocity" to explain why lead is a better projectile material than iron. This book is available free from Google Books.

"One draw-back on the theory of these gentleman, is their calculating the velocities with iron projectiles, for the heavier the material the more powerful the momentum, and consequently the longer the *retention of the velocity...*", my italics.

3. Input Data Used For Ballistic App Analysis

The input data used with each of the ballistic apps are summarized below. The baseline input for all were: air and powder temperature, 75F; ambient pressure, 27.52 mmHg; relative humidity, 50%, resulting density; 4,000 ft density altitude, 0.06786 lb/ft³; muzzle velocity, 3168 fps; sight height, 2.50 inch; zero range, 100 yds (first line of sight crossing).

The SmartShot data were supersonic velocity retention was 62.5, subsonic velocity retention was 142.0.

The following data show the adjustments used to force a path match to SmartShot at 1,000 yards.

Applied Ballistics 3.0: BCg7, 0.298; form factor, 0.912.

Ballistic Explorer V6.7.0.: BCg7, 0.298

Sierra Infinity 7: Velocity 3230 fps. Match was achieved with velocity change rather than change four BC's in different velocity bands.

Hornady 4DoF: 110 gn Sierra Match King, form factor, 1.08.

JBM: BCg7; BCg7, 0.300.

4. References

2.1 The bible for exterior ballistics is *Modern Exterior Ballistics* by Robert McCoy but it is physics and mathematics intense as well as hard to read for other reasons.

2.2 Damon Cali of Bison Ballistics has written an excellent summary of ballistics solvers in general and ballistic coefficients in specific; <https://bisonballistics.com/articles/the-details-of-a-ballistics-calculator-solver>. This site also has an excellent ballistics solver and a number of authoritative and interesting technical articles.

2.3 See *The Ballistic Coefficient* by John McDonald and Ted C. Almgren of the Sierra company for a fine detailed history and technical summary: http://www.exteriorballistics.com/ebexplained/articles/the_ballistic_coefficient.pdf.

2.4 Litz has excellent ballistic coefficient and shape factors information in his book, *Applied Ballistics for Long Range Shooting*, Chapter 2, The Ballistic Coefficient and Chapter 19, Using the Experimental Data. This is by far the best book on long range shooting I've seen. New long range shooters will learn faster as well as save the cost of this fine book many times over.

2.4 Wikipedia has some interesting information on the topic: https://en.wikipedia.org/wiki/Ballistic_coefficient.

2.5 A historical prospective of velocity retention appears in William Greener's book, *The Science of Gunnery As Applied To Military and Sporting Arms*, published in 1846, page 250, 251.

5. Disclaimers

All the usual disclaimers apply: 1. Don't play with guns because they might frighten someone, 2. To be safe stay on the couch (for safe couch behavior see rule 1.), 3.

ProjectileScience Inc. guarantees nothing, 4. Don't use SmartShot with real bullets, 5. Don't drink the printer ink (I didn't make this up: See the Canon G4210 printer safety instructions), and finally, 6. Don't do anything stupid.